Warning: Your solutions have to be substantiated, namely, you have to provide proofs for the given answers.

Recital Exercise 12.1 Provide the definition of each of the following terms: zonotope, projection (of a polytope), homogenization (of a hyperplane)

Recital Exercise 12.2 Prove that a polytope $P \subseteq \mathbb{R}^d$ is a zonotope if and only if there exists a natural number $k \geq 0$ and a matrix $A \in \mathbb{R}^{d \times k}$ such that $P = \{Ax \mid x \in C^k\}$. Here, C^k denotes the k-dimensional cube $[-1, 1]^k$.

Recital Exercise 12.3 Draw the 2-dimensional zonotope with generators

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1/2 \\ \sqrt{3}/2 \end{pmatrix}, \begin{pmatrix} -1/2 \\ -\sqrt{3}/2 \end{pmatrix} \in \mathbb{R}^2.$$

When you represent the zonotope as a projection of a k-dimensional cube for some k, which k do you choose and what is your matrix representing the projection?

Recital Exercise 12.4 Draw the 3-dimensional zonotope with generators

$$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1/2 \\ \sqrt{3}/2 \end{pmatrix}, \begin{pmatrix} 0 \\ -1/2 \\ -\sqrt{3}/2 \end{pmatrix}, \begin{pmatrix} 1/2 \\ 0 \\ 0 \end{pmatrix} \in \mathbb{R}^3.$$

Recital Exercise 12.5 Draw the 3-dimensional zonotope with generators

$$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \in \mathbb{R}^3.$$

Complementary Exercise 12.6 Let $\mathcal{A} = \{H_1, \ldots, H_n\}$ be a hyperplane arrangement in $\mathbb{R}^d$ where $H_i = \{x \in \mathbb{R}^d \mid a_i \cdot x = b_i\}$ for some $a_i \in \mathbb{R}^d$ and $b_i \in \mathbb{R}$ for every $i \in \{1, \ldots, n\}$. Define $H'_i = \{\binom{x}{t} \in \mathbb{R}^{d+1} \mid a_i \cdot x - b_i t = 0\}$ for every $i \in \{1, \ldots, n\}$, and consider the hyperplane arrangement $\mathcal{A}' = \{H'_1, \ldots, H'_n\}$. Prove that $\mathcal{V}^*(\mathcal{A}) = \mathcal{V}^*(\mathcal{A}')$.

Supplementary Exercise 12.7 Consider the zonotope in Recital Exercise 12.5. When you represent the zonotope as a projection of a k-dimensional cube for some k, which k do you choose and what is your matrix representing the projection?

Supplementary Exercise 12.8 Draw a 3-dimensional zonotope with generators

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} \in \mathbb{R}^3.$$

When you represent the zonotope as a projection of a k-dimensional cube for some k, which k do you choose and what is your matrix representing the projection?

Supplementary Exercise 12.9 Prove that a face of any zonotope is a translate of some zonotope.

Supplementary Exercise 12.10 Prove or disprove: For any $d \geq 1$, all d-dimensional zonotopes are simple.