Topics on Computing and Mathematical Sciences I	**July 9, 2008**
Graph Theory Lecture 12: Exercises	**Yoshio Okamoto**

Due Date: July 16, 2008
Legend: $(-)$ easy; $(+)$ hard
Warning: Your solutions have to be substantiated, namely, you have to provide proofs for the given answers.

Exercise 12.1 $(-)$ Prove that $R(k,\ell) \leq \binom{k+\ell-2}{k-1}$ for $k,\ell \geq 1$. (Hint: $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$.)

Exercise 12.2

1. Prove $R(3,4) \leq 9$. (Hint: First prove $R(3,4) \leq 10$. Then examine the proof carefully to deduce $R(3,4) \leq 9$.)
2. Prove $R(4,4) \leq 18$.

Exercise 12.3

1. Prove that $R(P_4, C_4) = 5$.
2. Prove that $R(C_4, C_4) = 6$.

Exercise 12.4 Let k, ℓ be natural numbers such that $k-1$ divides $\ell-1$. Prove that $R(T, K_{1,\ell}) = k+\ell-1$ if T is any k-vertex tree.

Exercise 12.5 Prove that $R(k_1,\ldots,k_m) \leq 2-m+\sum_{i=1}^{m} R(k_1,\ldots,k_i-1,\ldots,k_m)$. Use this to derive $R(k_1,\ldots,k_m) \leq \dfrac{(k_1+\cdots+k_m-m)!}{(k_1-1)!\cdots(k_m-1)!}$.