Due Date: June 18, 2008
Legend: (−) easy; (+) hard
Warning: Your solutions have to be substantiated, namely, you have to provide proofs for the given answers.

Exercise 8.1 (−) Let G be a planar graph that contains no K_3, and $n(G) \geq 3$. Prove that $e(G) \leq 2n(G)-4$.

Exercise 8.2 A graph is *outerplanar* if it has a planar embedding with every vertex on the boundary of the unbounded face. Prove that neigher K_4 nor $K_{2,3}$ is outerplanar.

Exercise 8.3 Let G be an n-vertex planar graph with girth k, where k is finite. Prove that G has at most $(n-2)\frac{k}{k-2}$ edges Use this to prove that the Petersen graph is nonplanar.

Exercise 8.4 Prove that every planar graph with at least four vertices has at least four vertices with degree less than six. For each even value of n with $n \geq 8$, construct an n-vertex simple planar graph G that has exactly four vertices with degree less than six.

Exercise 8.5 (+) Prove that a maximal planar graph is 3-colorable if and only if it is Eulerian. (Hint: For sufficiency, use induction on $n(G)$. Choose an appropriate pair of triple of adjacent vertices to replace with appropriate edges.)

Exercise 8.6 Using Kuratowski's theorem, prove that every planar graph contains no K_5-minor or no $K_{3,3}$-minor.