Due Date: June 11, 2008
Legend: (−) easy; (+) hard
Warning: Your solutions have to be substantiated, namely, you have to provide proofs for the given answers.

Exercise 7.1 Prove that if G is a k-critical graph, then the graph $M(G)$ generated from G by applying Mycielski's construction is k+1-critical.

Exercise 7.2 (−) Prove that every 3-regular Hamiltonian graph is 3-edge-colorable.

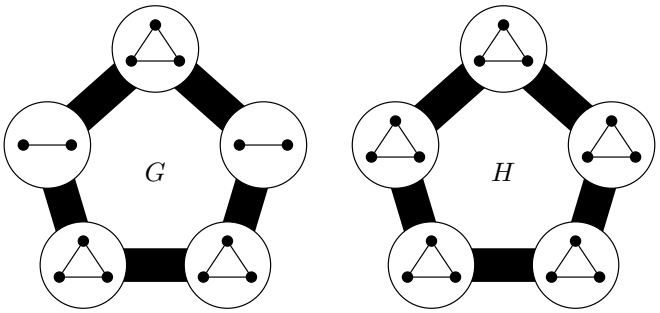
Exercise 7.3 Let $k \geq 3$, and G be a k-regular graph with a cut-vertex. Prove that $\chi'(G) > k$.

Exercise 7.4 Prove that the complement of the line graph of a bipartite graph is perfect.

Exercise 7.5 (−)

1. Find a K_4-subdivision in the Grötzsch graph.
2. Prove that every r-chromatic graph contains a K_r-subdivision for $r = 2, 3$.

Exercise 7.6 Thick edges below indicate that every vertex in one circle is adjacent to every vertex in the other. Prove that $\chi(G) = 7$ but G has no K_7-subdivision. Prove that $\chi(H) = 8$ but H has no K_8-subdivision.



Exercise 7.7 Denote by $\chi''(G), \chi_\ell(G), \chi'_\ell(G)$ the total chromatic number, the list chromatic number, the list chromatic index of G, respectively.

1. Prove that $\Delta(G) + 1 \leq \chi''(G) \leq 2\Delta(G)$ for every graph G. (Hint: For the upper bound, consider applying Brooks' theorem.)
2. Prove that $\chi_\ell(G) \leq \Delta(G)$ and $\chi'_\ell(G) \leq 2\Delta(G) - 1$.
3. Prove that $\chi''(G) \leq \chi'_\ell(G) + 2$.