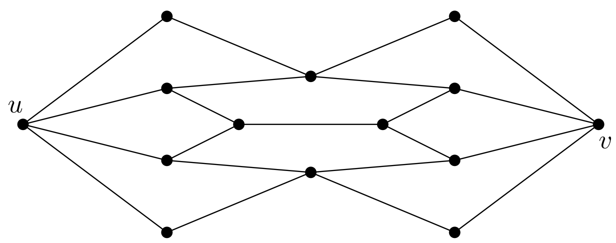
Due Date: May 21, 2008
Legend: (−) easy; (+) hard
Warning: Your solutions have to be substantiated, namely, you have to provide proofs for the given answers.

Exercise 5.1 For each choice of integers k, ℓ, m with $0 < k \leq \ell \leq m$, construct a graph G with $\kappa(G) = k$, $\kappa'(G) = \ell$ and $\delta(G) = m$. Remember to justify your construction.

Exercise 5.2 Prove that a graph is 2-vertex-connected if and only if it has at least three vertices and for every ordered triple (x, y, z) of distinct vertices of G has an x, z-path through y.

Exercise 5.3 Prove that if $G = (V, E)$ is a non-complete 2-vertex-connected k-regular graph, where $k \geq 3$, then G has three distinct vertices $x, y, z \in V$ such that $\{x, y\}, \{x, z\} \in E$, $\{y, z\} \notin E$ and $G - \{y, z\}$ is connected. (Note: This exercise will be used in the next lecture as a lemma.)

Exercise 5.4 (−) Determine $\kappa(u, v)$ and $\kappa'(u, v)$ in the graph drawn below.



Exercise 5.5 Prove that $\kappa(G) = \kappa'(G)$ when G is 3-regular. (Hint: You may use Menger's theorem, but not necessarily.)

Exercise 5.6 Let n be even and G be an n-vertex r-vertex-connected graph having no $K_{1,r+1}$ as an induced subgraph. Prove that G has a perfect matching. (Hint: Use Tutte's theorem on the existence of a perfect matching.)

Exercise 5.7 For a graph G and a natural number $k \geq 0$, prove that if $n(G) \geq k+1$ and $\delta(G) \geq \lceil (n+k-2)/2 \rceil$, then G is k-vertex-connected.