Vertex Angle and Crossing Angle Resolution of Leveled Tree Drawings

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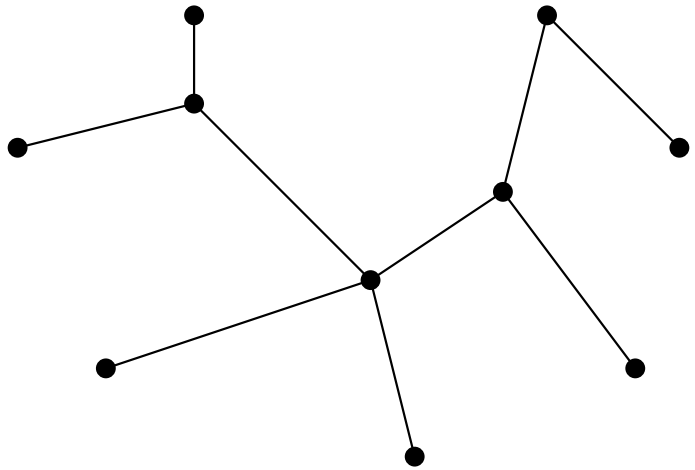
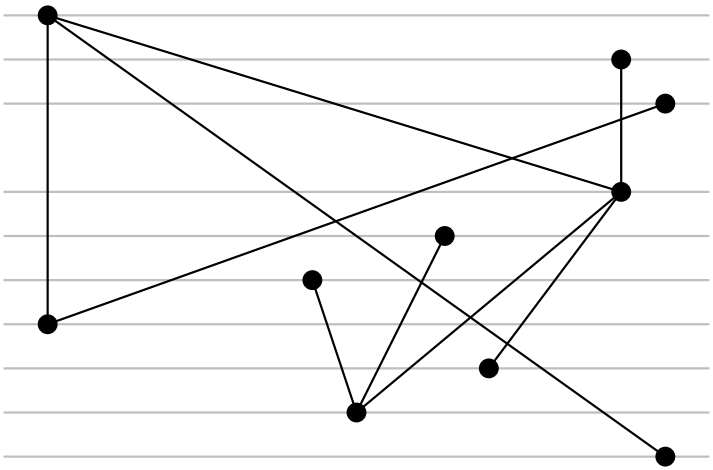
Drawing trees

A tree is a connected graph without cycle



Every tree can be drawn on the plane without edge crossing

How about if the y-coordinates are fixed?



Def.: Leveled tree

A leveled tree is a pair (T, L) of

- a tree T and
- an injection $L\colon V \to \mathbb{R}$

A leveled drawing of (T, L) is a straight-line drawing of T s.t. for every vertex $v \in T(V)$ the y-coordinate of v is $L(v)$

Drawing leveled trees

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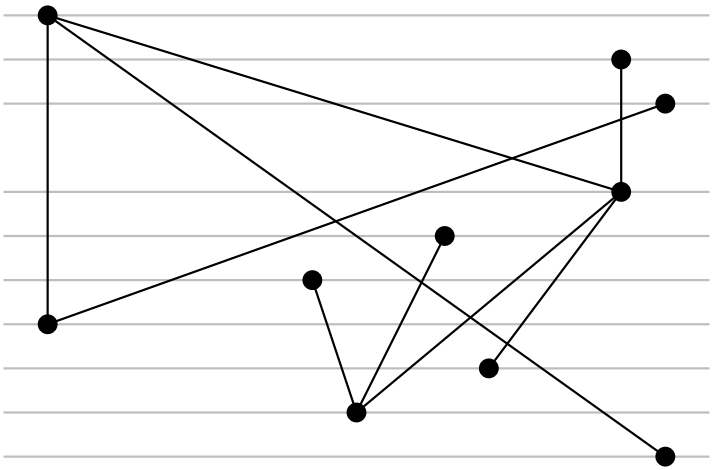

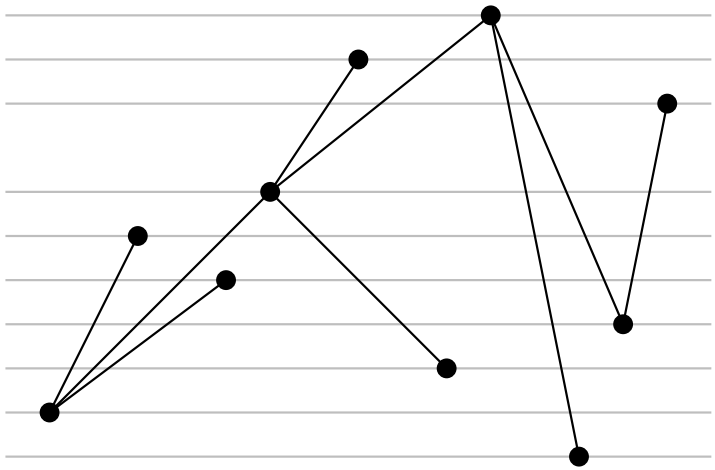

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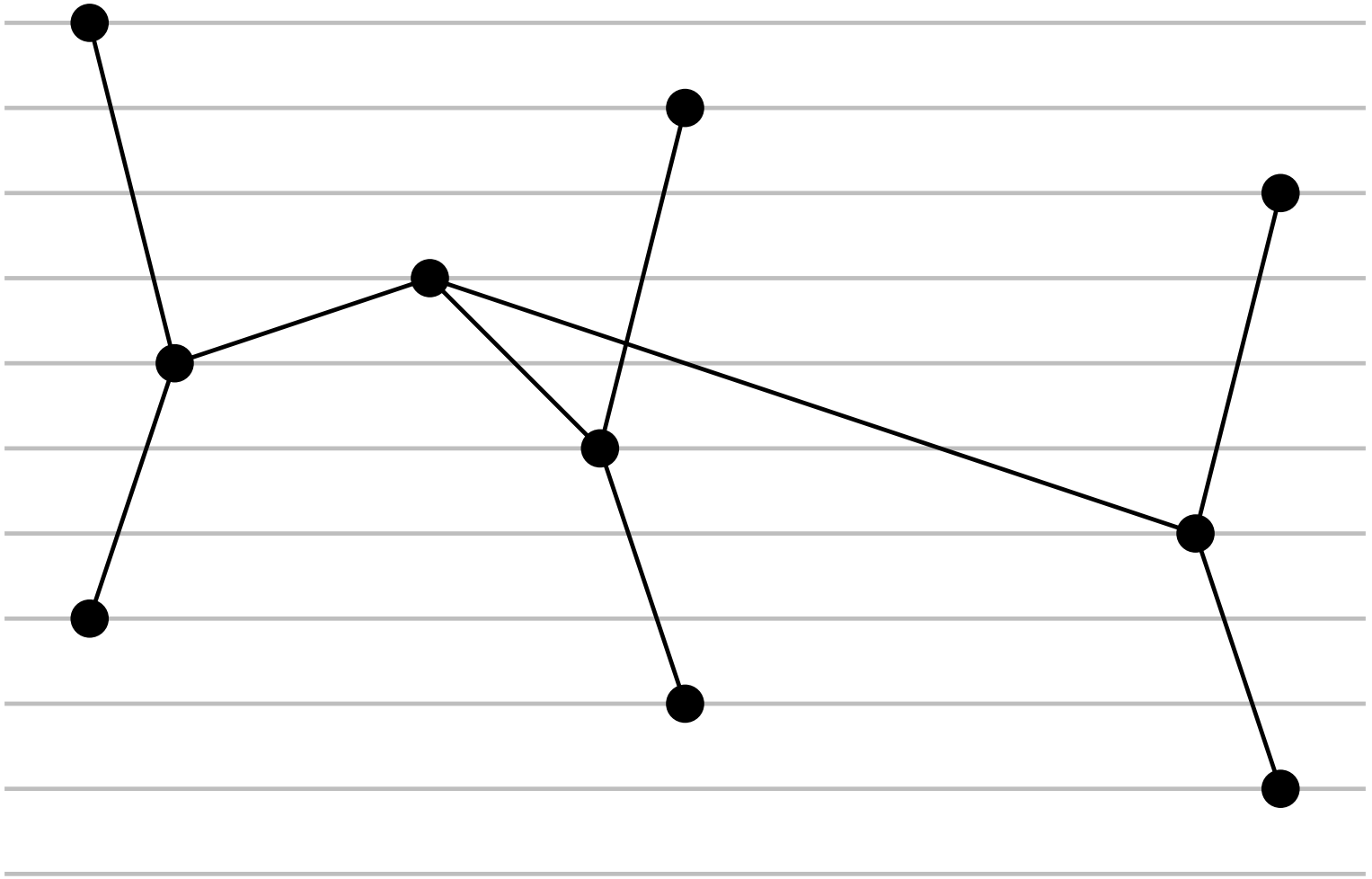
A non-planar example of leveled trees

Not all leveled trees can be drawn without edge crossing



[Healy, Kuusik, Leipert '04]

Even though edges cross, drawings should be nice

Reflection on aesthetics

- Not every (leveled) graph can be drawn without edge crossing
- $\therefore$ Imposing "planarity" is too restricted
- $\therefore$ Need to live with crossings

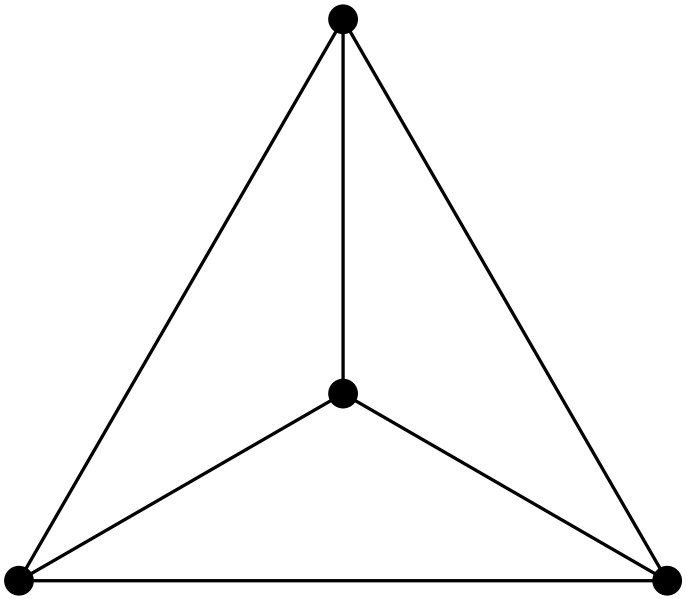
Cognitive experiments (Huang '07, Huang, Hong, Eades '08)

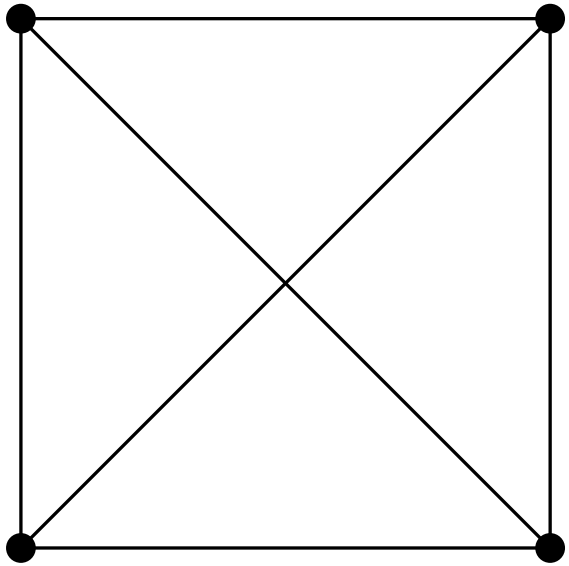
- The readability of non-planar drawing is severely affected by those crossings that form "sharp" angles
- Crossing angles of at least 70° have smaller impact on human task performance

- $\therefore$ A drawing with large crossing angles is desired

Example: K_4

A crossing doesn't harm the readability if it creates a large angle





Another example

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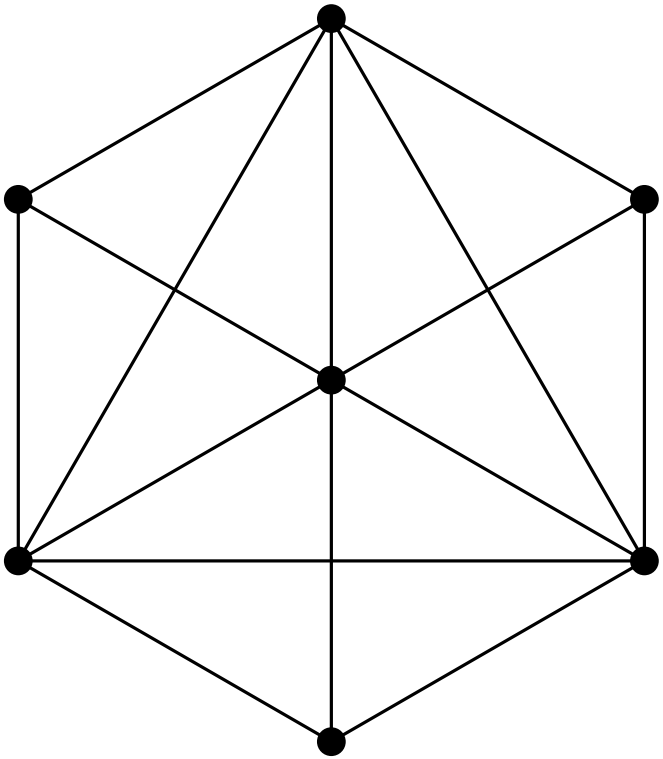
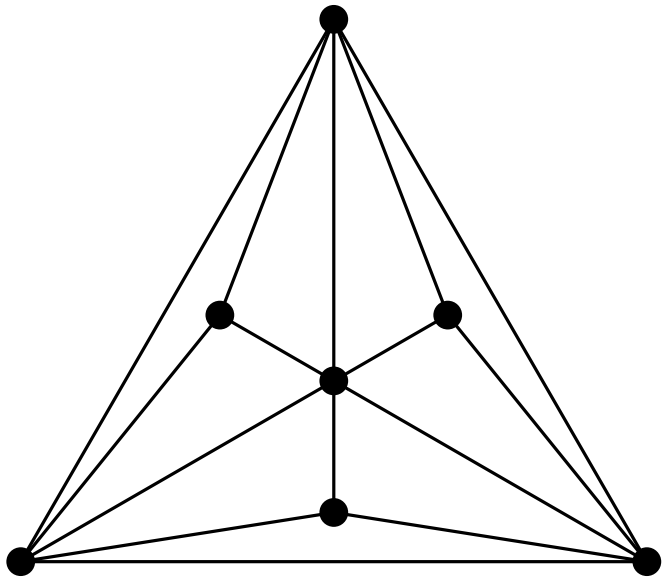
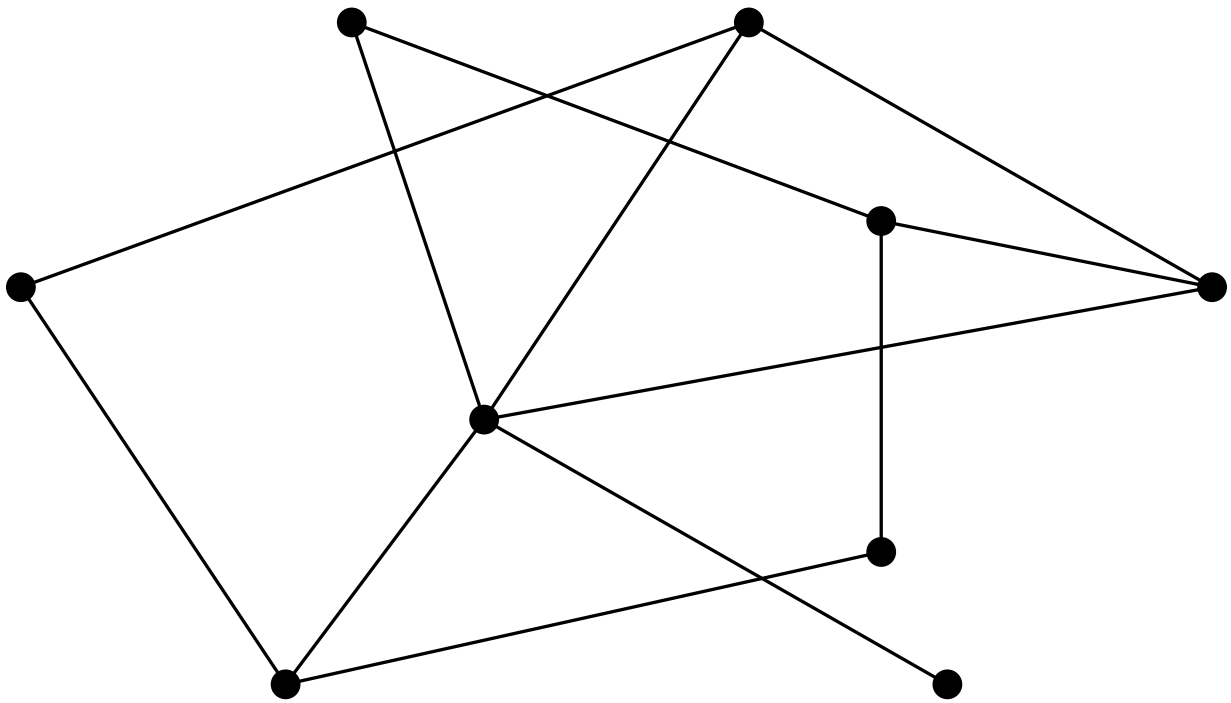




The crossing angle resolution of a drawing is ...

the minimum angle formed by a crossing in the drawing



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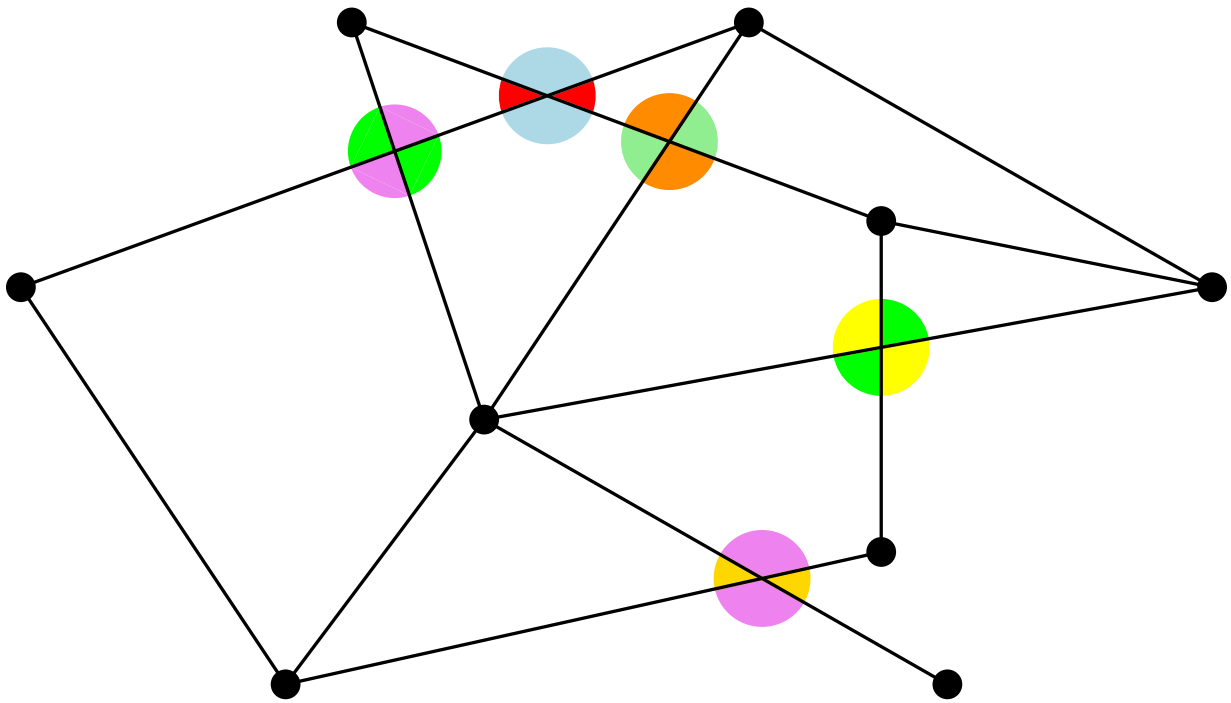
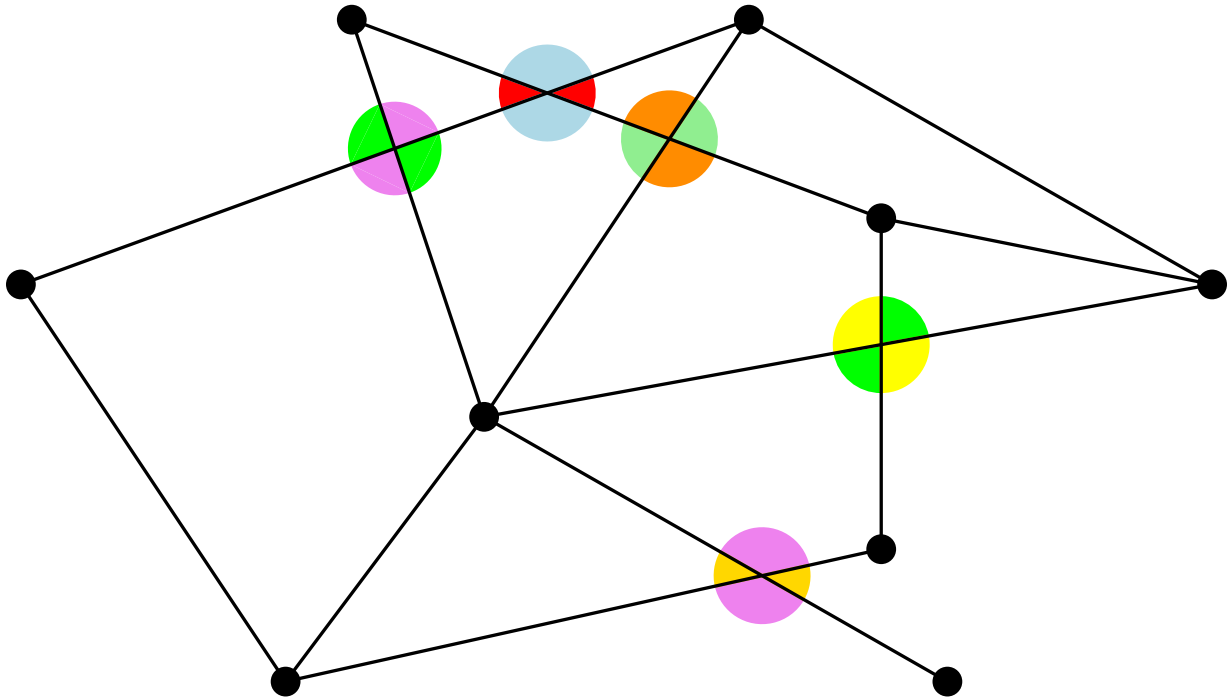


Our task

Given a (leveled) graph
Find a (leveled) drawing with large crossing angle resolution



Theorem 1

$\forall$ leveled tree (T, L)
$\exists$ a leveled drawing of (T, L) with
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Remark: $\exists$ a leveled tree (T, L) such that
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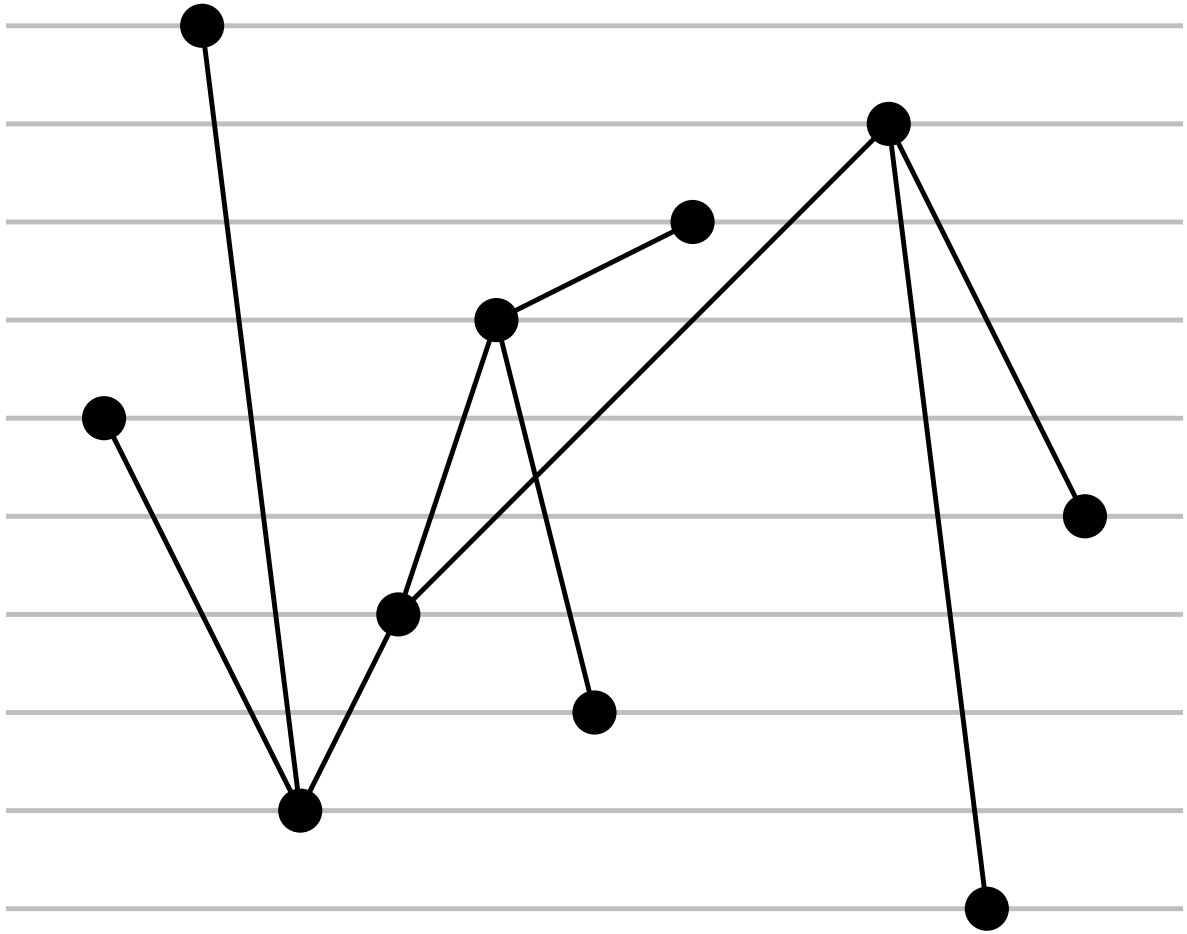
Algorithm for Theorem 1 (1/2)

First step

Given a leveled tree



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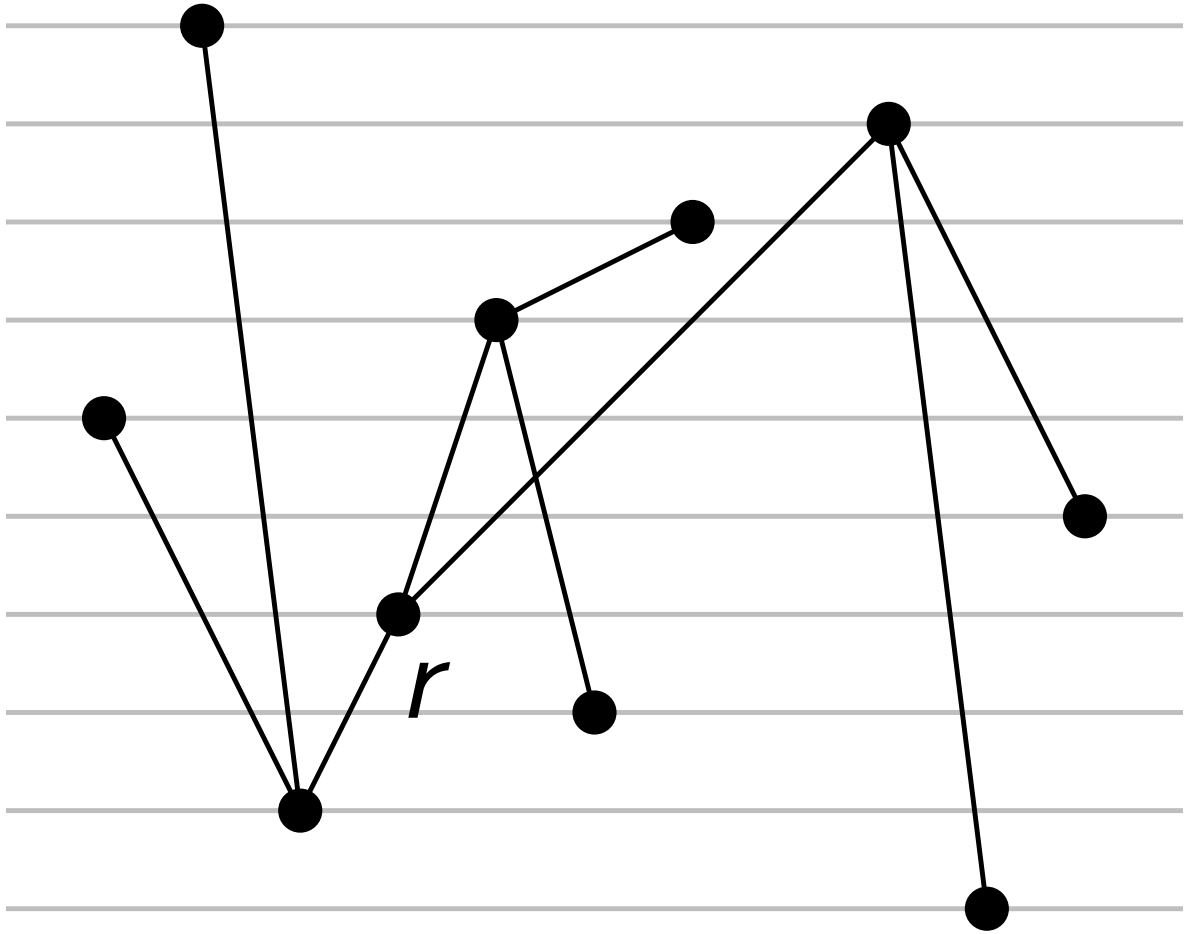
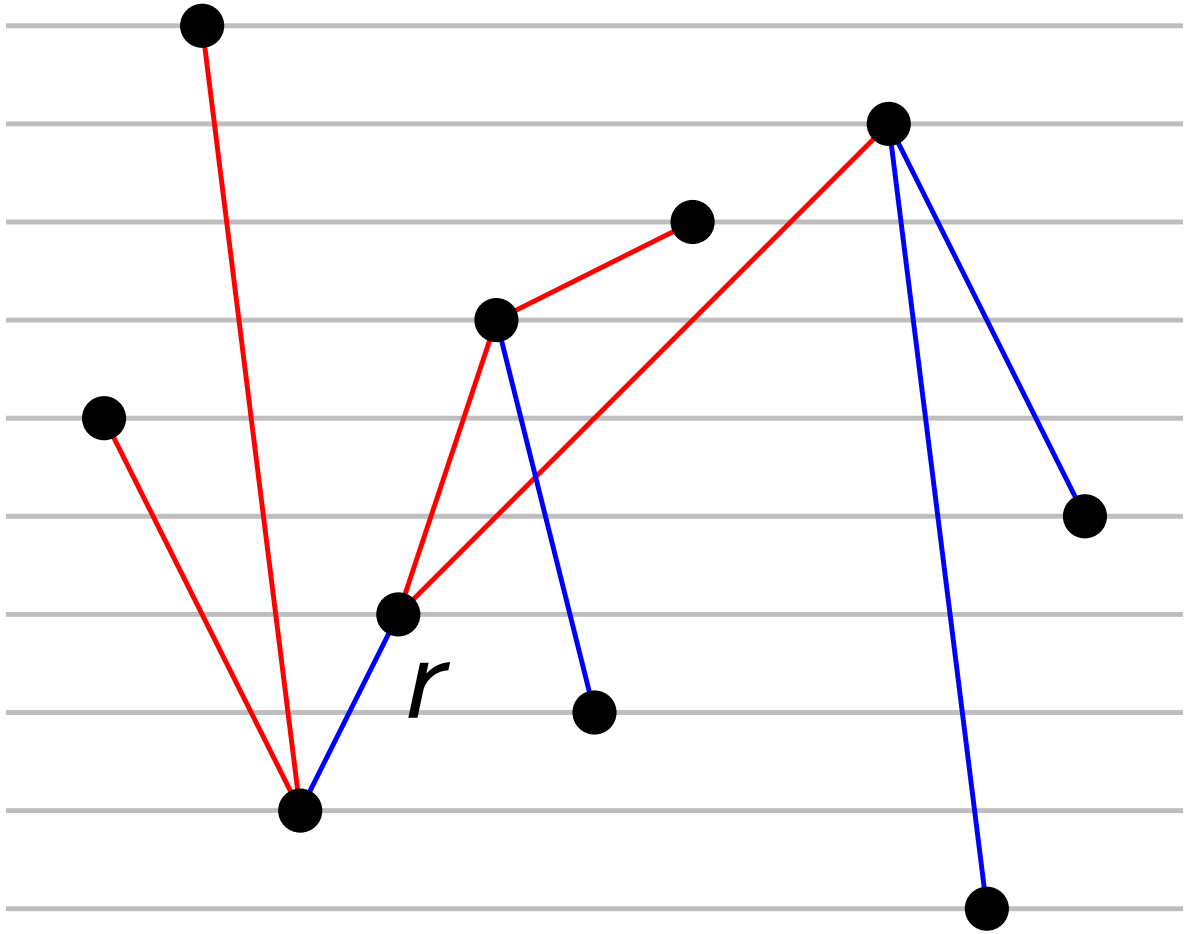
We fix a root arbitrarily



Algorithm for Theorem 1 (1/2)

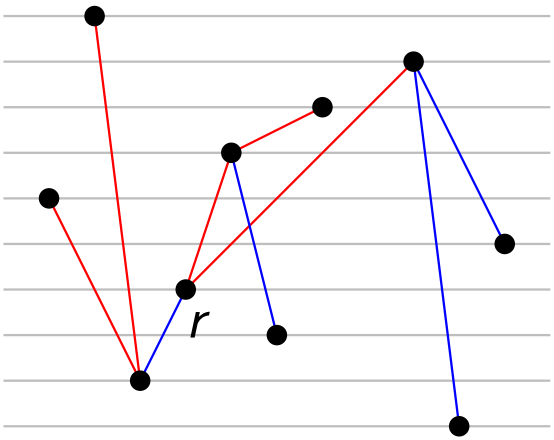
First step

Color an edge red if the parent is lower than the child, blue o/w



Second step

We place red edges w/ slope 45° and blue edges w/ slope -45° from the root to the leaves (with a sufficiently small perturbation)



Algorithm for Theorem 1 (2/2)

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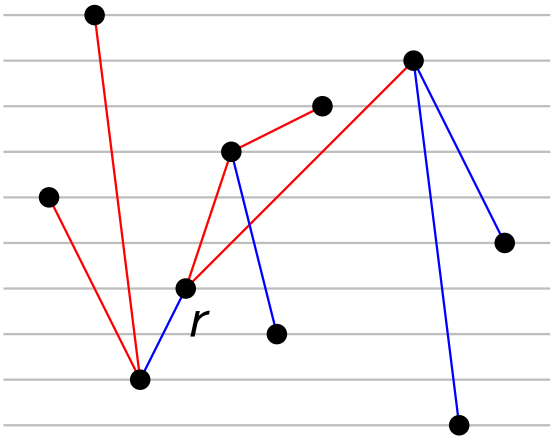

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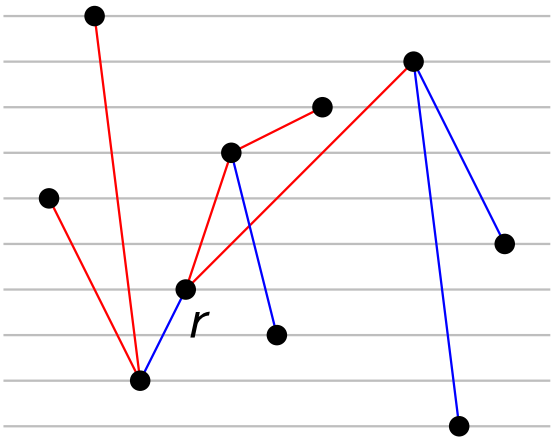

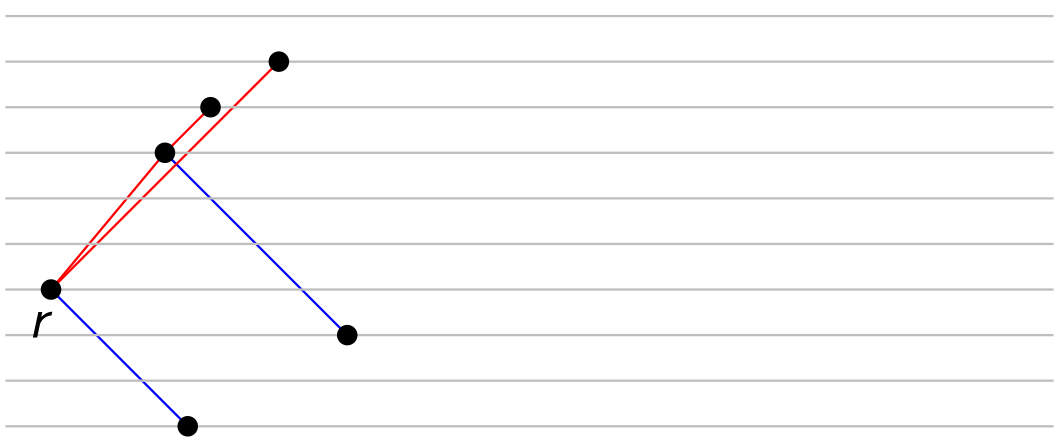

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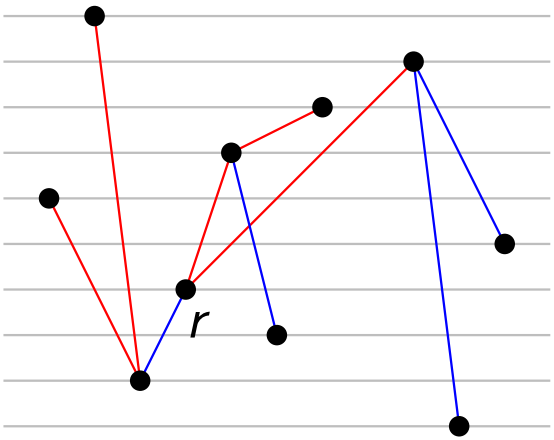

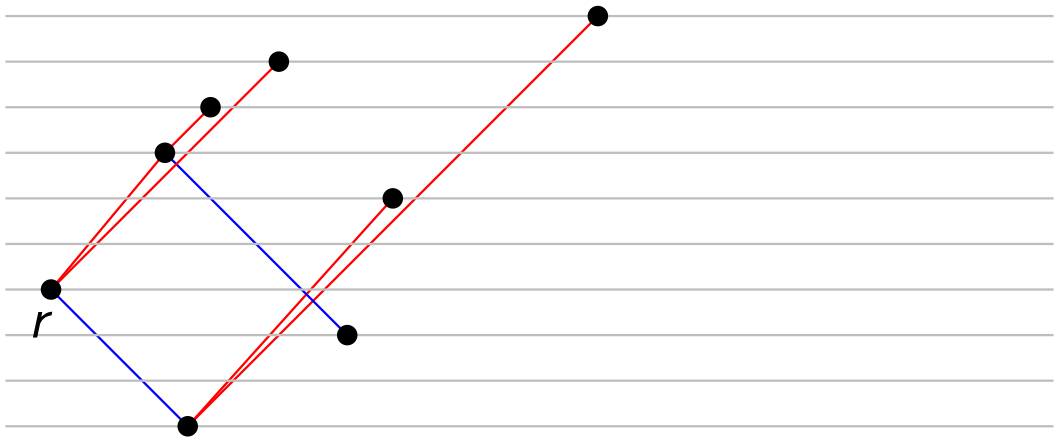

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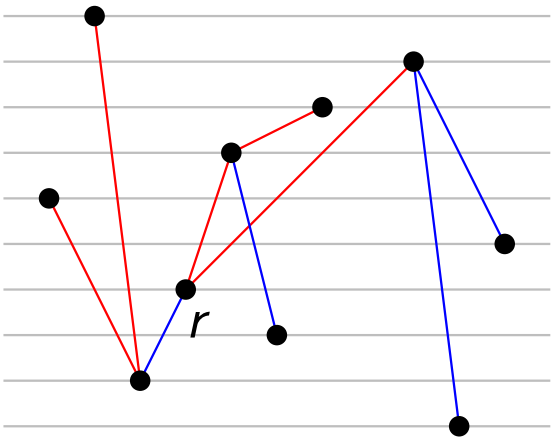

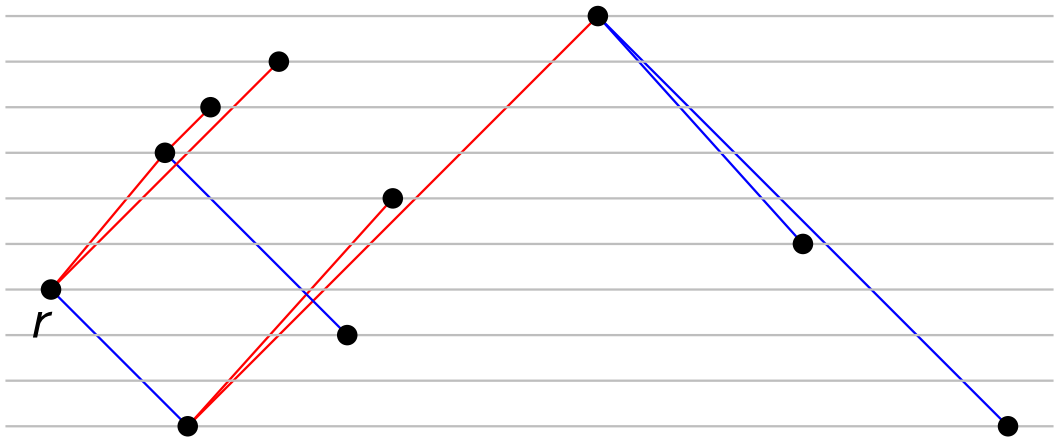

What is the crossing angle resolution of such drawings?

At least $90° - \epsilon$ since

- a crossing occurs between a red edge and a blue edge, and
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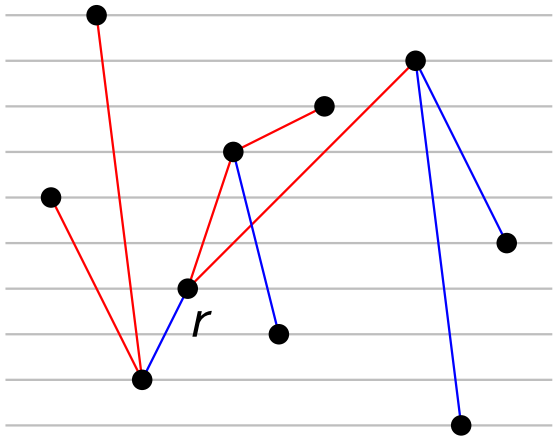

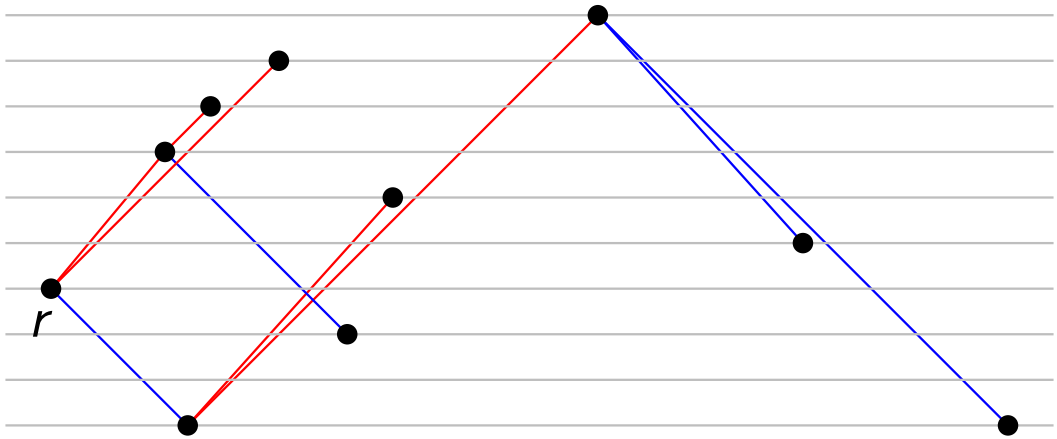

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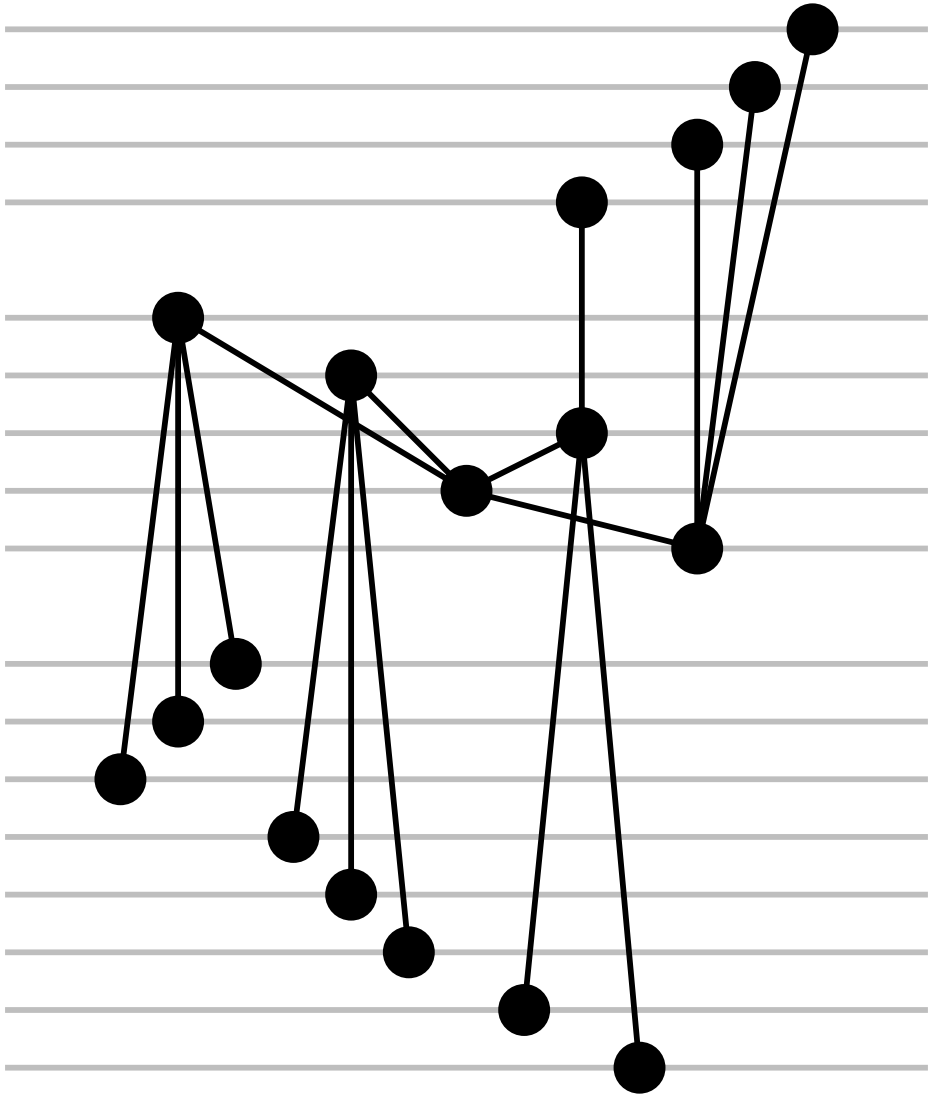
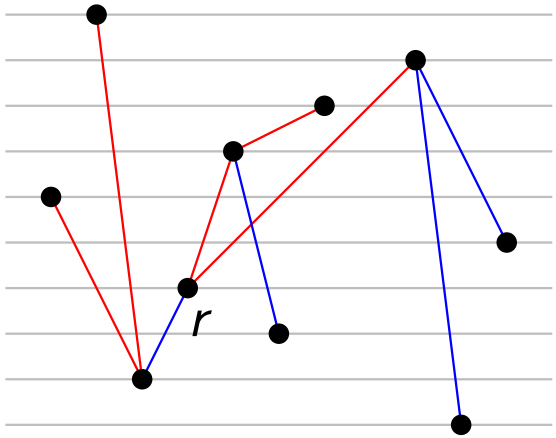
Upper bound construction for Result 1

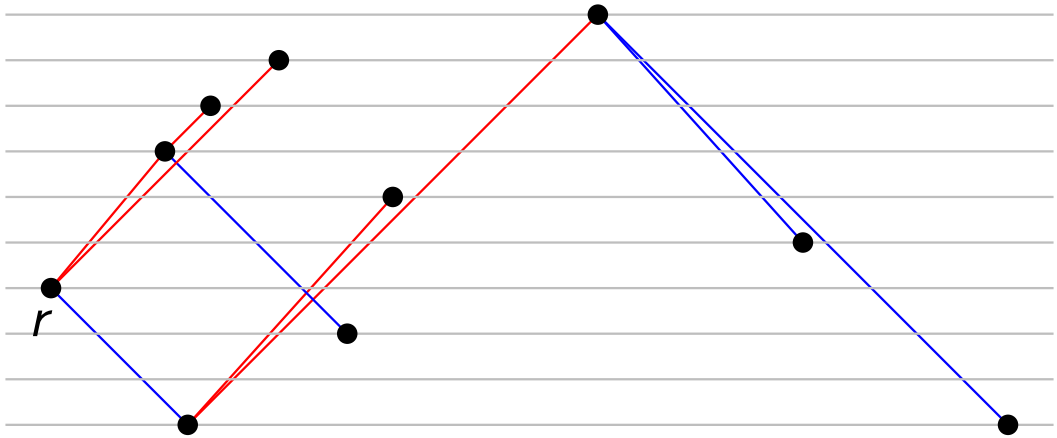
For this leveled tree, the crossing angle resol'n $< 90°$



Our drawing is unsatisfactory!!

Because of too small angles around vertices



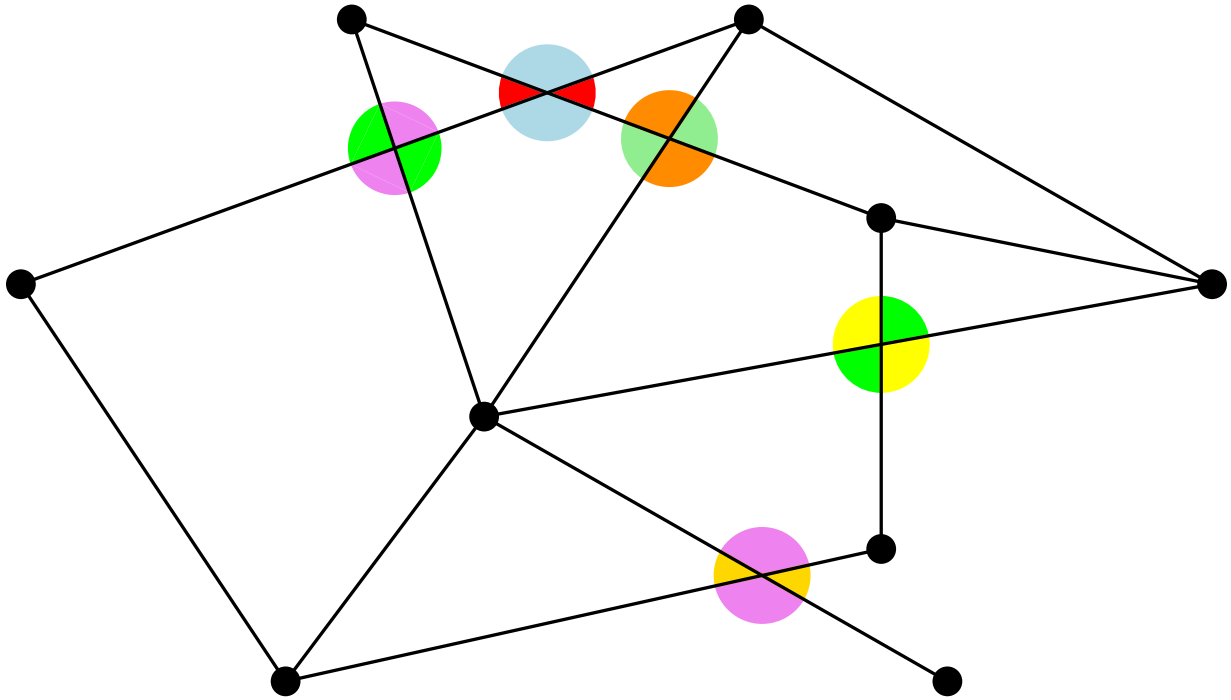


We need to take care of the angles around vertices, too!

Total angle resolution

The total angle resolution of a drawing is ...

the minimum angle formed by a crossing or two incident edges in the drawing



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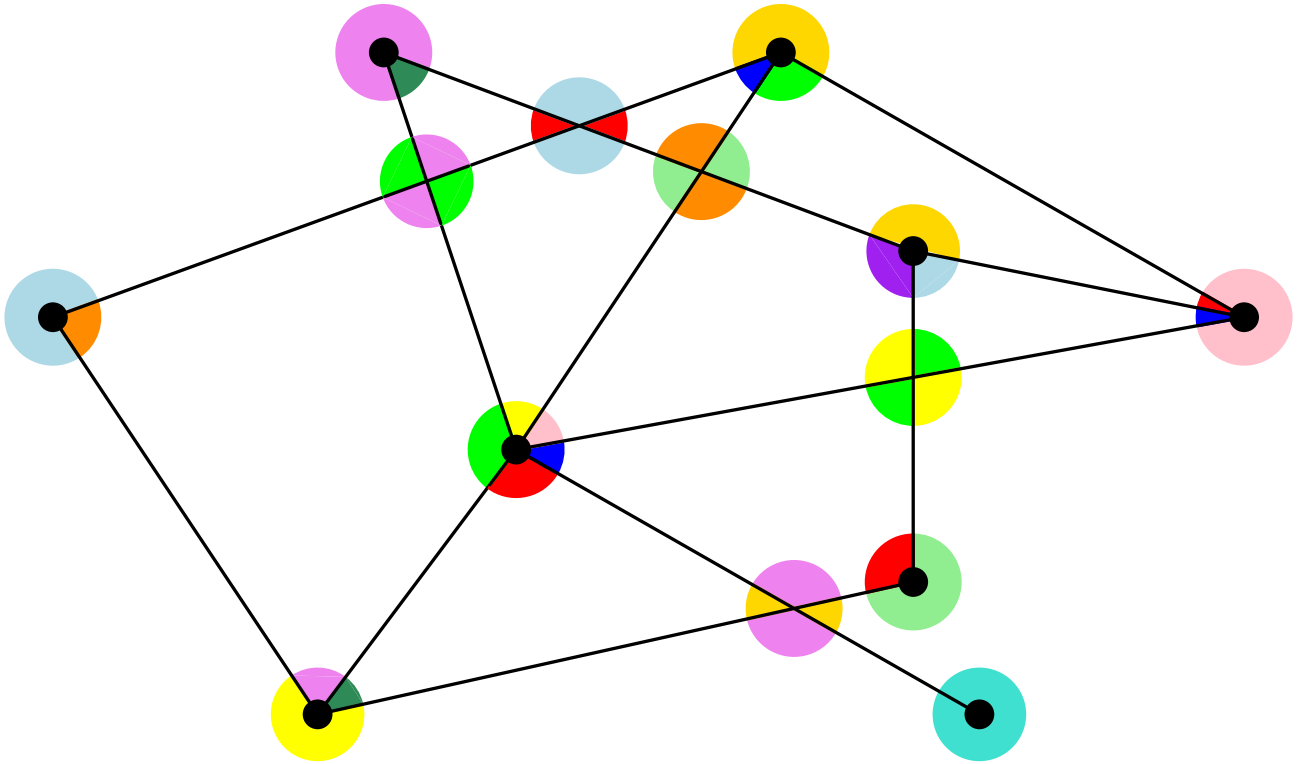

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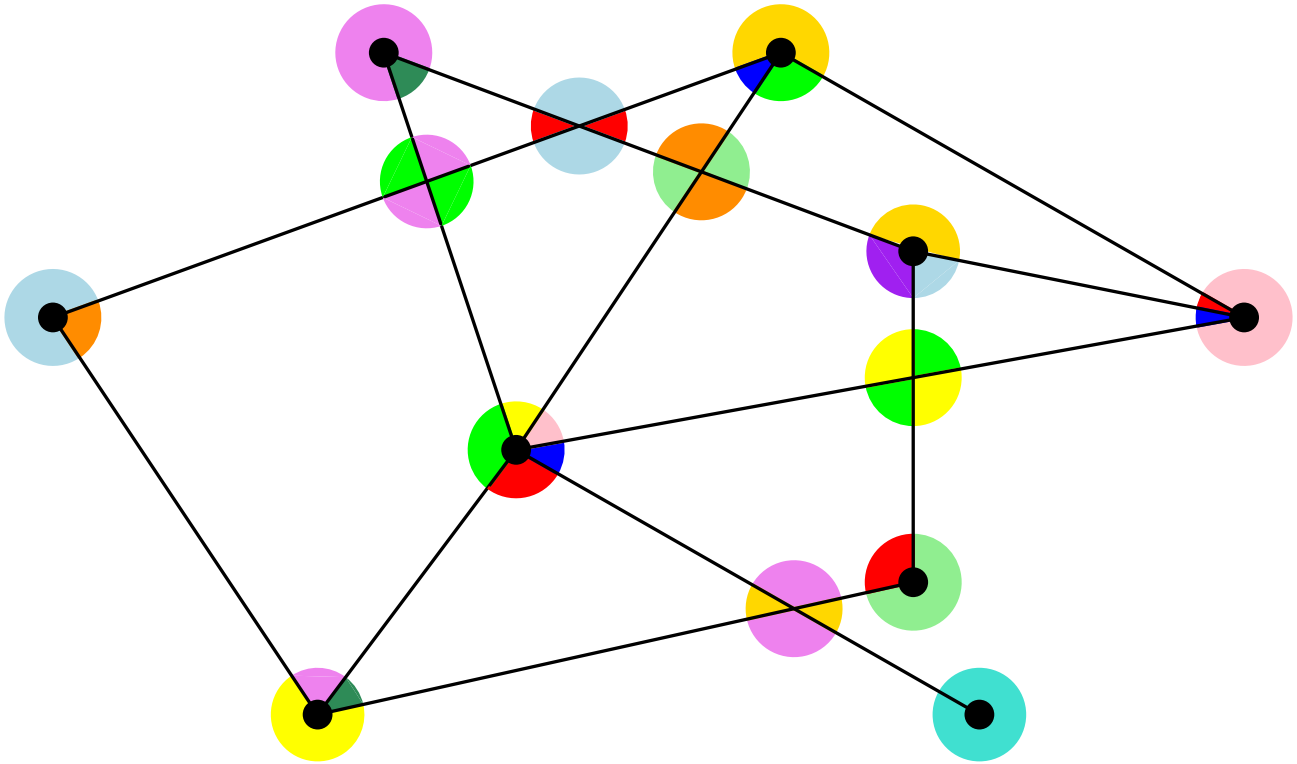

Theorem 2

$\forall$ leveled tree (T, L)
$\exists$ a leveled drawing of (T, L) with
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where $d =$ max degree of T

Remark: $\exists$ a leveled tree (T, L) such that
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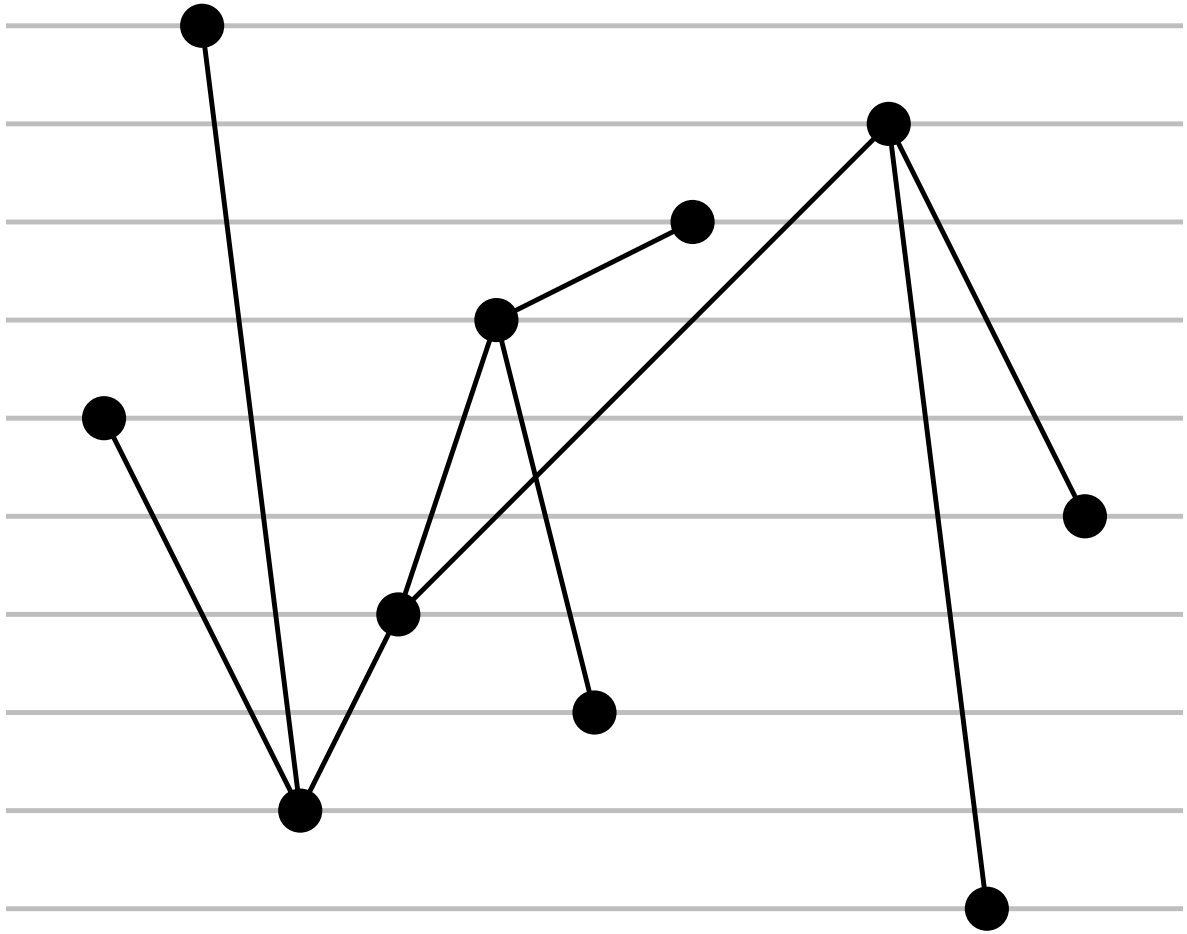
Algorithm for Theorem 2 (1/2)

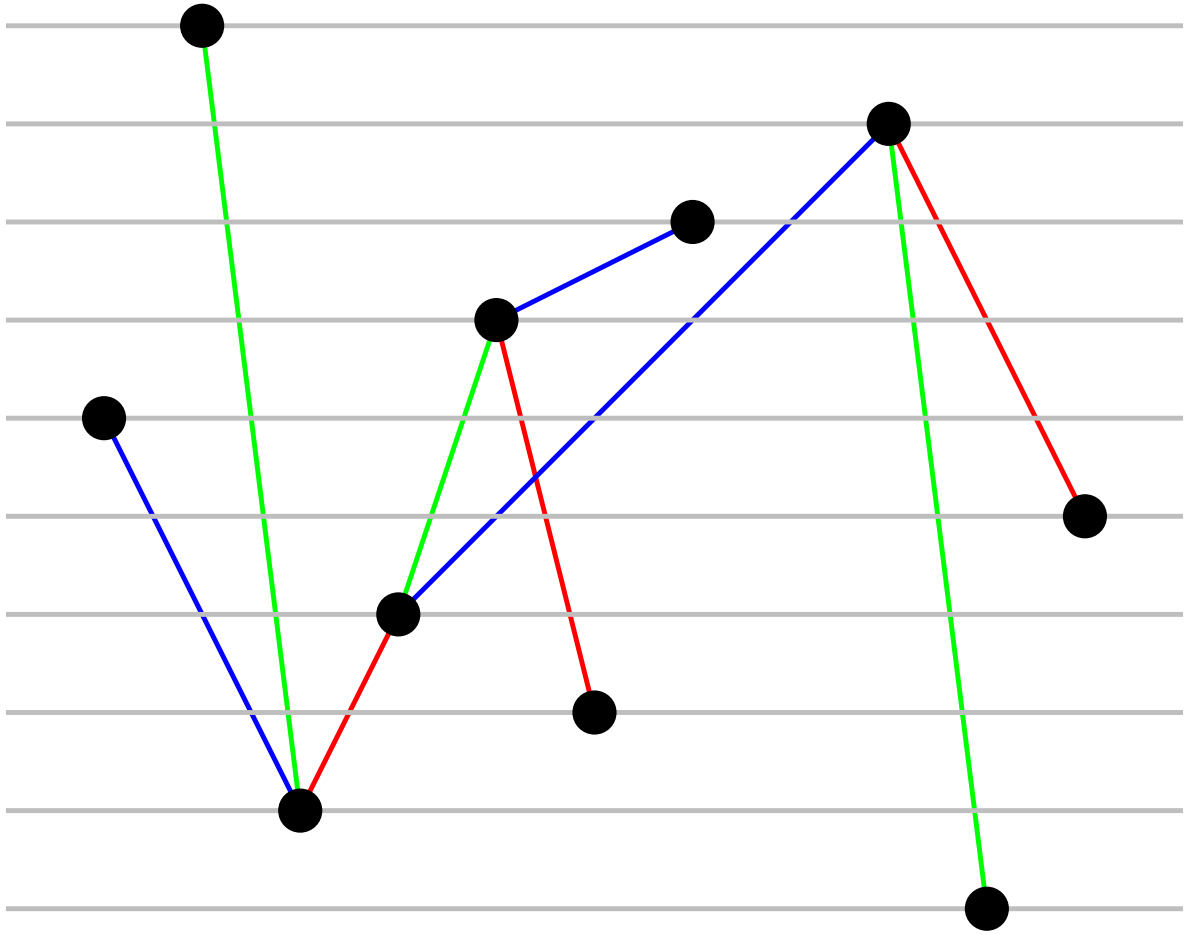
First step

Given a leveled tree



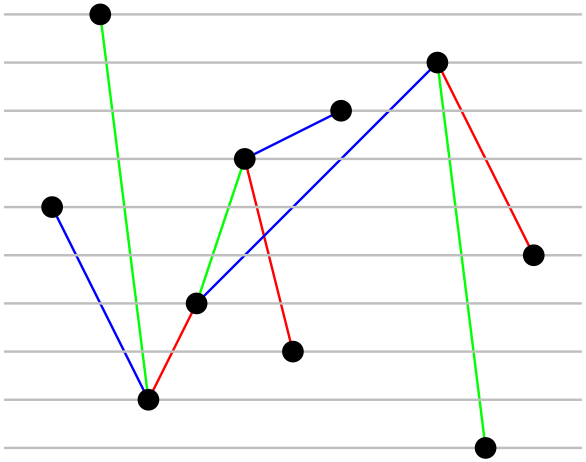
First step

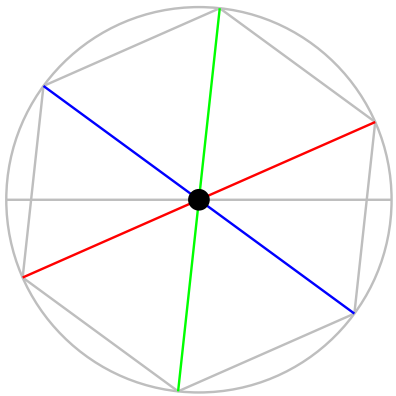
We fix a proper d-edge-coloring arbitrarily



Second step

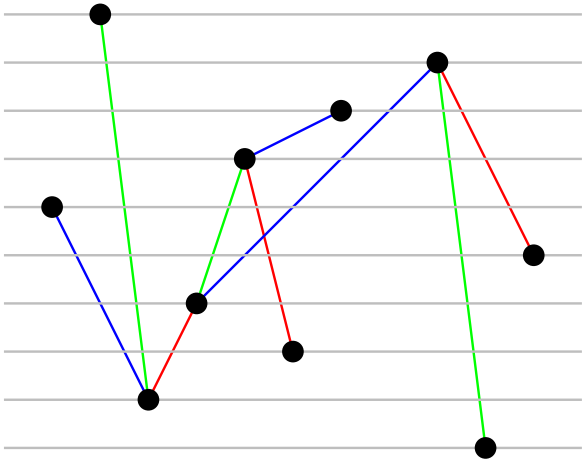
Consider the $2d$-gon w/ long diagonals colored by d colors

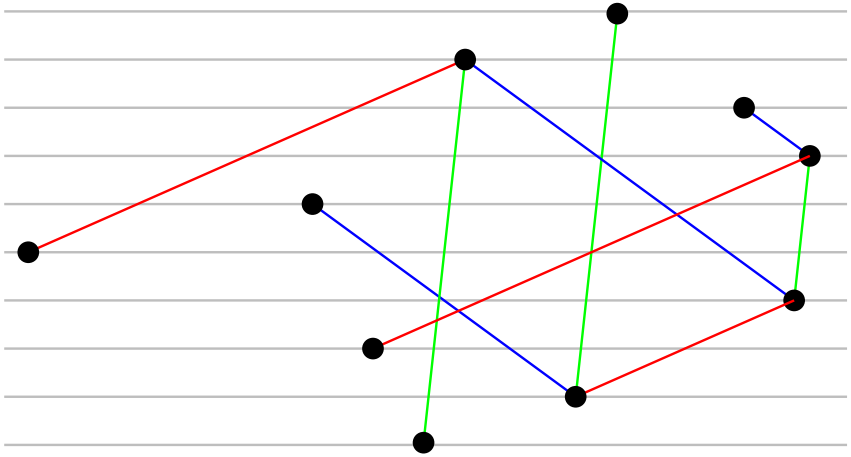


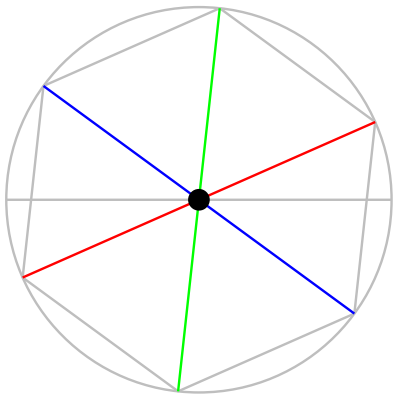


Second step

Draw the edges of T with the slope of the same-color diagonals







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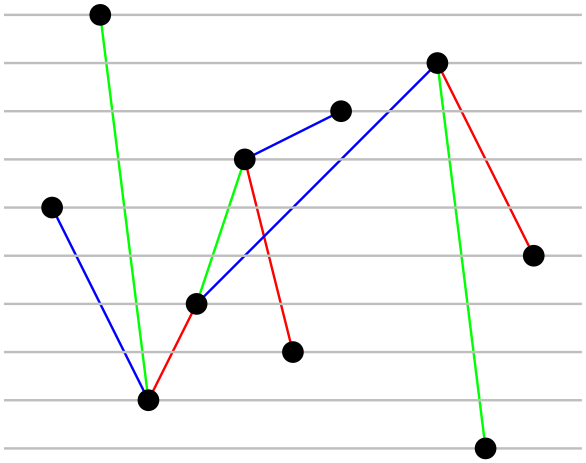

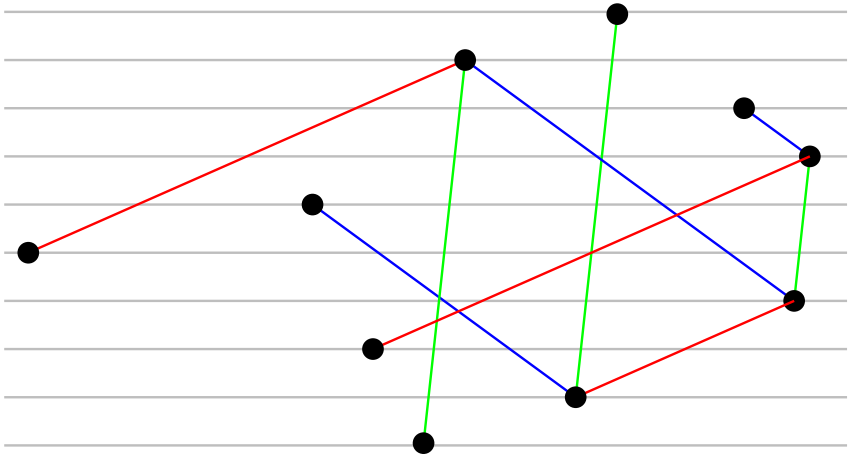

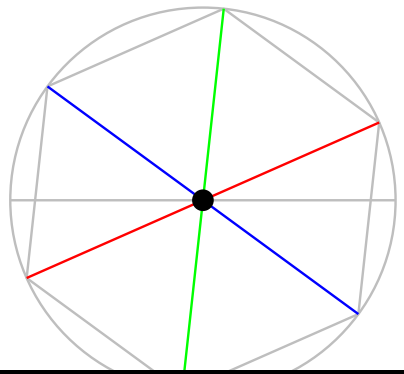

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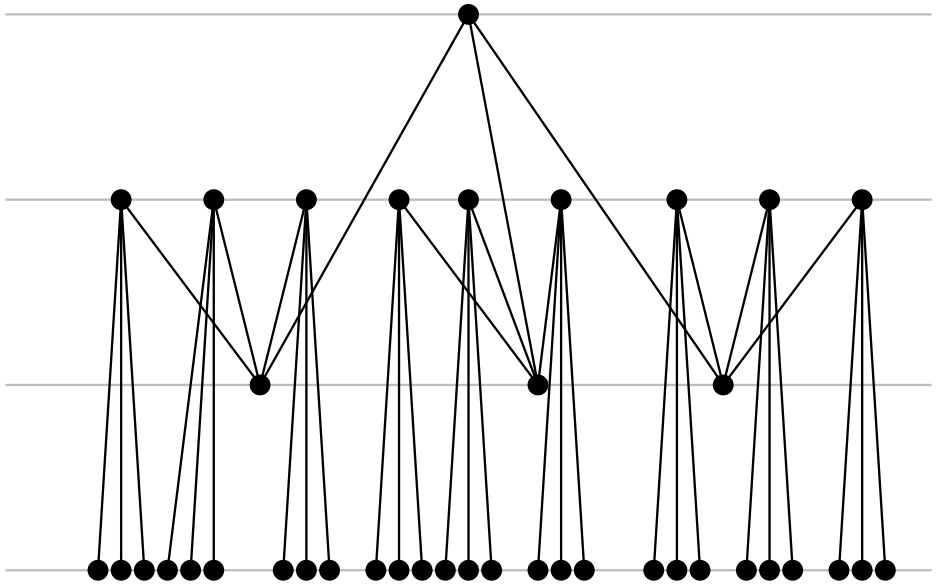
Upper bound construction for Result 2

For this leveled tree, the total angle resol'n $\leq \pi/d$

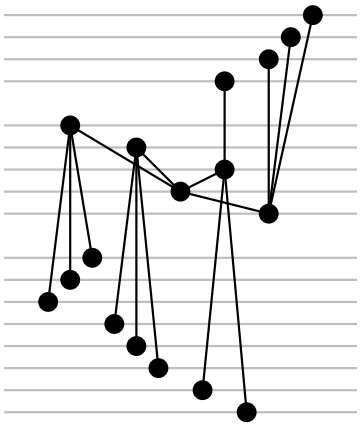


(when $d = 3$)

Summary

- Tight bounds of
  - the crossing angle resolution $(\pi/2 - \epsilon)$
  - the total angle resolution $(\pi/d - \epsilon)$

  in leveled tree drawings
- Question: The crossing angle resolution when $d = 3$?
- Question: What about leveled planar graphs?



[Thank you]